ZARGON ANKAN

All questions carry equal marks, try as much as possible

1. Let $ABCD$ be a trapezium in which $AD \parallel BC$. Also $BA \perp AD$. Now, $ABCD$ has an incircle which touches BC at P and AD at R. Given that length of $PC = 36$ and length of $RD = 49$, find the length PR. (you can refer to the figure below)
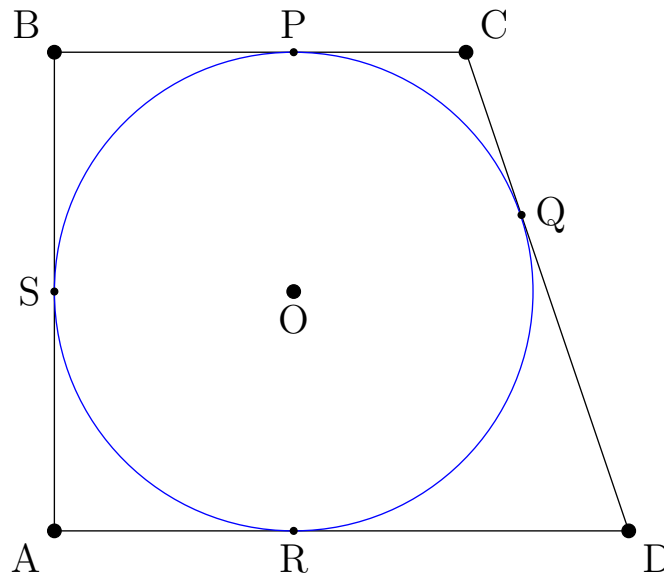
2. Let x and y be positive real numbers such that $x = 1 - y$. Prove that

$$x^x y^y + x^y y^x \leq 1.$$

3. Let there exist real numbers a, b, c such that $a^2 + 4b^2 + 16c^2 = 48$ and $ab + 4bc + 2ca = 24$. Then find the value of $a^2 + b^2 + c^2$?

4. Let x, y, $z > 0$ and $xyz = 1$. Then prove that

$$\frac{1 + xy}{1 + x} + \frac{1 + yz}{1 + y} + \frac{1 + zx}{1 + z} \geq 3$$

5. Show that given a subset of $n+1$ elements of $\{1, 2, 3, \ldots, 2n\}$, there are two elements in that subset such that one divides the other.

6. We have $x, y, k, n \in \mathbb{N}$. Prove that $k|n$ iff $(x^k - y^k)|(x^n - y^n)$.

End